Caringbah High School

2017

Trial HSC Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen (Black pen is preferred)
- Board-approved calculators may be used
- A Board-approved reference sheet is provided for this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

60 marks

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1) What is the remainder when $2x^3 - 10x^2 + 4$ is divided by $x - 2$?

A) -52

B) -20

C) $x^3 - 5x^2 + 2$

D) $-x^3 + 5x^2 - 2$

2) The point P divides the interval from $A(-2,2)$ to $B(8,-3)$ internally in the ratio 3:2. What is the x-coordinate of P?

A) 4

B) 2

C) 0

D) -1

3) Which expression is equivalent to $\tan\left(\frac{\pi}{4} - x\right)$?

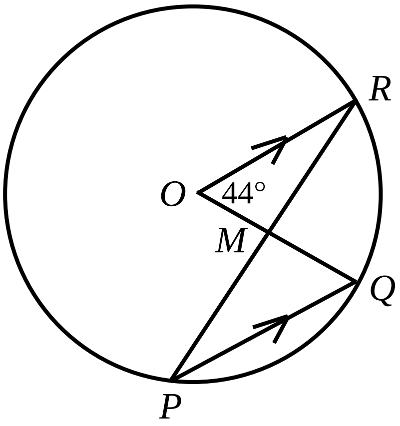
A) $1 - \tan x$

B) $1 + \tan x$

C) $\dfrac{\cos x - \sin x}{\cos x + \sin x}$

D) $\dfrac{\cos x + \sin x}{\cos x - \sin x}$

4)



NOT TO SCALE

O is the centre of the circle.

OR is parallel to *PQ*.

$\angle ROM = 44°$.

What is the size of $\angle RMQ$?

A) 22°

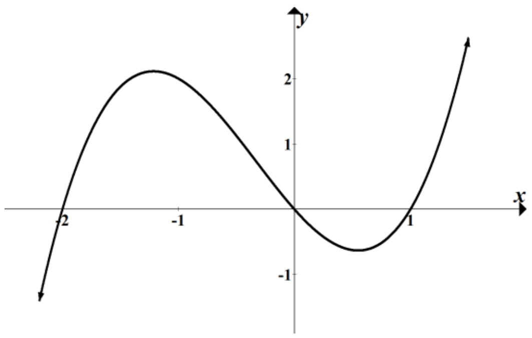
B) 44°

C) 66°

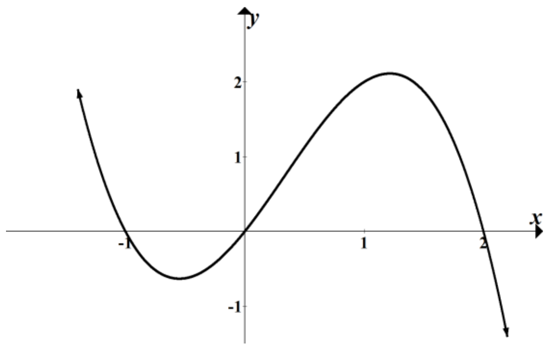
D) 88°

5) Which of the following could be the graph of $y = 2x - x^2 - x^3$?

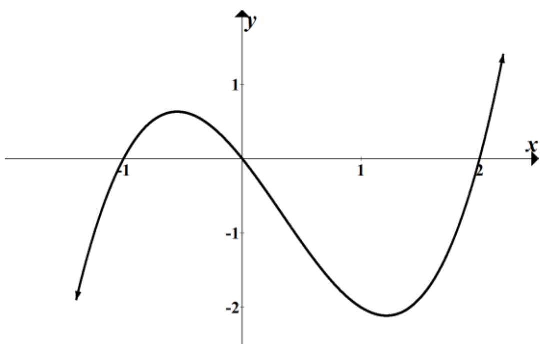
A)



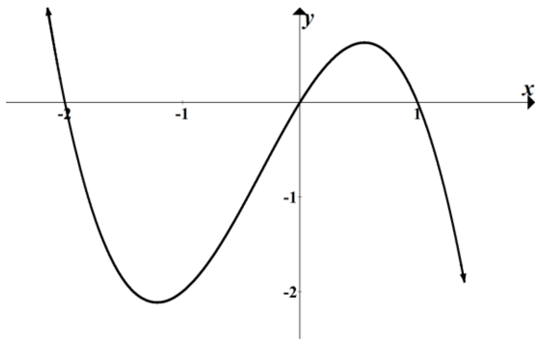
B)



C)



D)



6) What are the asymptotes of the curve defined by the parametric equations $x = \frac{1}{t},\ y = \frac{t}{t+1}$?

A) $x = -1,\ y = 0$

B) $x = 1,\ y = -1$

C) $x = 0$ only

D) $x = -1$ only

7) To the nearest degree, what is the acute angle between the lines $2x + y = 5$ and $x - 3y = 1$?

A) 18°

B) 45°

C) 72°

D) 82°

8) What is the value of k such that $\int_0^{2k} \frac{1}{\sqrt{3 - x^2}}\, dx = \frac{\pi}{3}$?

A) $\frac{\sqrt{3}}{4}$

B) $\frac{\sqrt{3}}{2}$

C) $\frac{3}{4}$

D) $\frac{3}{2}$

9) Consider the function $f(x) = x^3 - 12x$.

What is the largest possible domain containing the origin for which $f(x)$ has an inverse function $f^{-1}(x)$?

A) $-2 < x < 2$

B) $-2 \le x \le 2$

C) $-2\sqrt{3} < x < 2\sqrt{3}$

D) $-2\sqrt{3} \le x \le 2\sqrt{3}$

10) The acceleration of a particle is defined in terms of its position by $\ddot{x} = 2x^3 + 4x$.

The particle is initially 2 metres to the right of the origin, travelling at $6\,\text{ms}^{-1}$.

What is the minimum speed of the particle?

A) $36\,\text{ms}^{-1}$

B) $20\,\text{ms}^{-1}$

C) $16\,\text{ms}^{-1}$

D) $6\,\text{ms}^{-1}$

END OF MULTIPLE CHOICE QUESTIONS

Section II

60 marks
Attempt all questions 11−14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11−14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a NEW booklet. **Marks**

a) Find $\frac{d}{dx}\left[\log_e(\ln x)\right]$. **1**

b) Show that $\lim_{\theta \to 0} \frac{5\theta}{\tan\frac{\theta}{2}} = 10$. **2**

c) If $y = be^{ax}$ find $\frac{d^2y}{dx^2}$ in terms of y. **2**

d) Find $\int \cos^2 2x \, dx$. **2**

e) Prove $\frac{2\cos\theta}{\operatorname{cosec}\theta - 2\sin\theta} = \tan 2\theta$. **2**

f) Solve $\frac{x^2 - 3}{2x} \geq 0$. **3**

g) Solve $e^t - e^{-t} = 2$, expressing the answer in simplest exact form. **3**

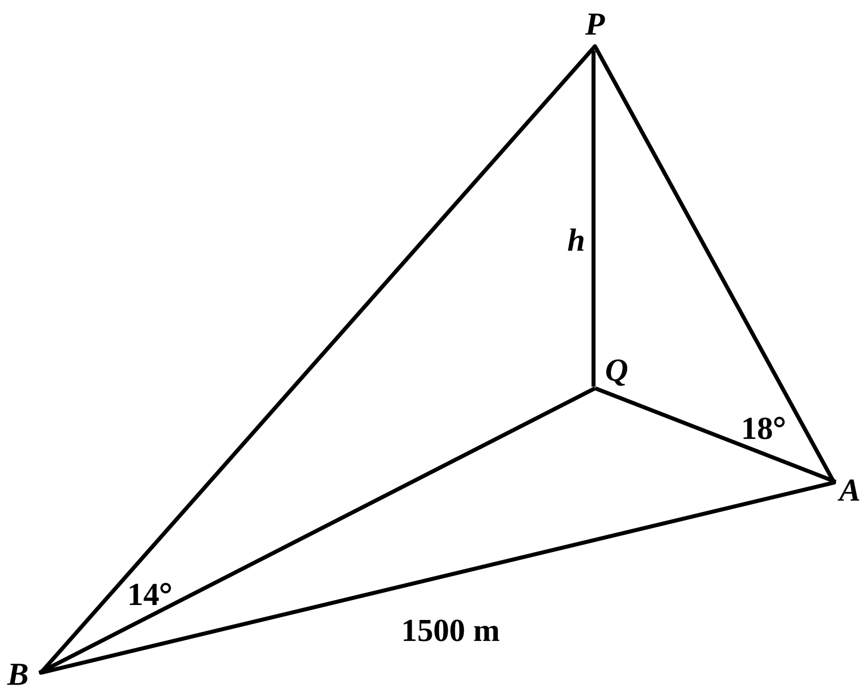
Question 12 (15 marks) Start a NEW booklet. **Marks**

a) i) Express $\sin\theta - \sqrt{3}\cos\theta$ in the form $A\sin(\theta - \alpha)$, where $0 \le \alpha \le \frac{\pi}{2}$. **2**

ii) Hence, or otherwise, find the minimum value of $\sin\theta - \sqrt{3}\cos\theta$. **1**

b) Use the substitution $u = x^4 - 1$ to evaluate $\int_0^1 \frac{x^3}{1 + x^8}\, dx$. **3**

c) The angle of elevation of a tower *PQ* of height *h* metres at a point *A* due east of it is 18°. From another point *B*, due south of the tower the angle of elevation is 14°. The points *A* and *B* are 1500 metres apart on level ground.



i) Show that $BQ = h\tan 76°$. **1**

ii) Find the height *h* of the tower to the nearest metre. **2**

Question 12 continues on page 8

Question 12 (continued) **Marks**

d) i) Show that $P(x) = 3\sin^{-1}\left(\frac{x}{2}\right) - 2x$ is an odd function. **1**

ii) Carefully sketch $y = 3\sin^{-1}\left(\frac{x}{2}\right)$ and $y = 2x$ on the same number plane. **2**

iii) It is known that a close approximation to a root of $P(x) = 0$ is $x_1 = 1.9$. **2**

Use one application of Newton's method to find a better approximation x_2. Give your answer correct to 3 decimal places.

iv) Find the sum of the roots of $P(x) = 0$. **1**

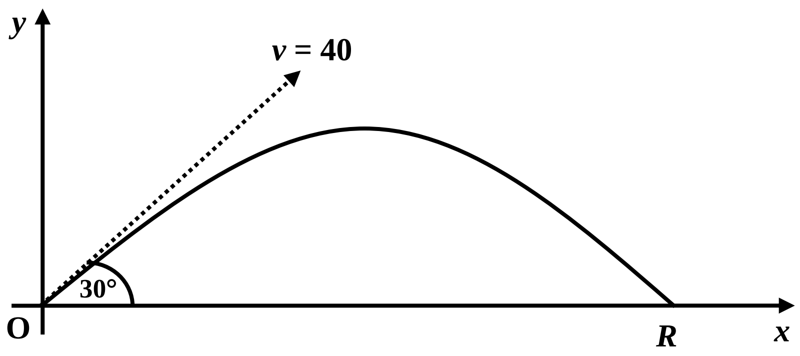
End of Question 12

Question 13 (15 marks) Start a NEW booklet. **Marks**

a) Use mathematical induction to prove that $13 \times 6^n + 2$ is divisible by 5 for all integers $n \geq 1$. **3**

b) A particle is projected with an initial velocity of 40 m s^{-1} at an angle of $30°$ to the horizontal. The equations of motion are given by

$$x = 20\sqrt{3}\,t, \quad y = 20t - 5t^2. \quad \text{(Do NOT prove this.)}$$



i) Find the maximum height of the particle. **2**

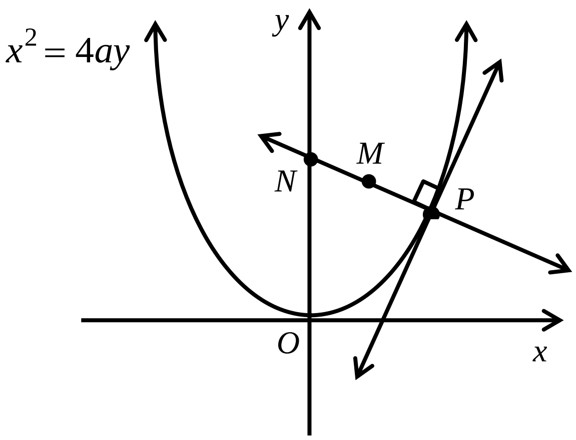
ii) Find the range (R) of the particle. **1**

iii) Determine the cartesian equation of the particles flight path. **1**

Question 13 continues on page 10

Question 13 (continued) **Marks**

c) $P\left(2ap, ap^2\right)$ is a point on the parabola $x^2 = 4ay$. The normal to the parabola at P cuts the y-axis at N. M is the midpoint of PN.



i) Show that N has coordinates $\left(0, ap^2 + 2a\right)$. **1**

ii) Show that the locus of M as P moves on the parabola $x^2 = 4ay$ is another parabola and state its focal length. **3**

d) The velocity v m s^{-1} of a particle moving in simple harmonic motion along the x-axis is given by $v^2 = 5 + 4x - x^2$.

i) Between which two points is the particle oscillating? **1**

ii) What is the amplitude of the motion? **1**

iii) Find the acceleration of the particle in terms of x. **1**

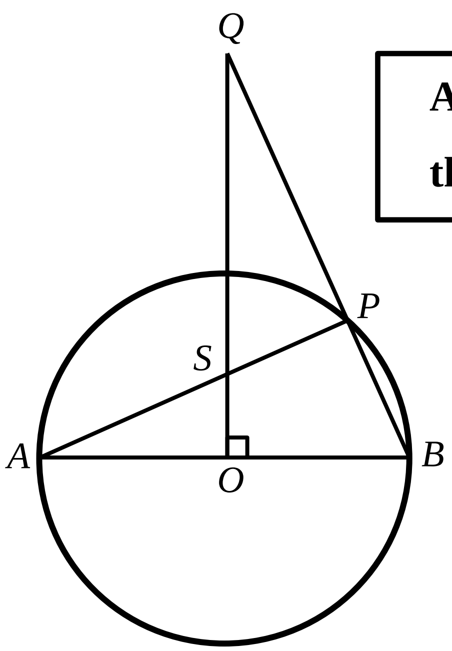
iv) Find the period of oscillation. **1**

End of Question 13

Question 14 (15 marks) Start a NEW booklet. **Marks**

a) The circle centred at O has a diameter AB. BPQ and ASP are straight lines.

Also, the line OSQ is perpendicular to AOB.



Answer this question on the page provided

i) Show that $AOPQ$ is a cyclic quadrilateral. **2**

ii) Show that $\angle AQO = \angle OBS$. **2**

b) The roots α, β and γ of the equation $2x^3 + 9x^2 - 27x - 54 = 0$ are consecutive terms of a geometric sequence.

i) Show that $\beta^2 = \alpha\gamma$. **1**

ii) Find the value of $\alpha\beta\gamma$. **1**

iii) Find the roots α, β and γ. **3**

Question 14 continues on page 12

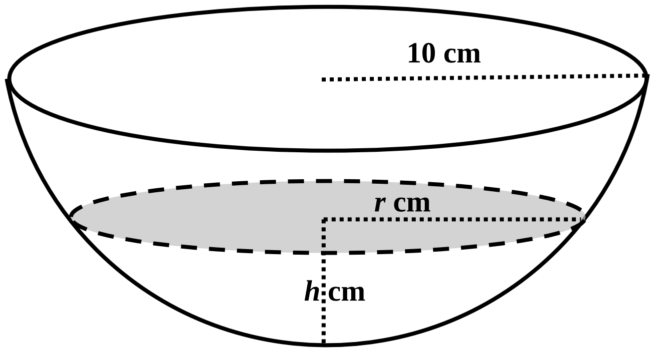
Question 14 (continued) **Marks**

c) Let a hemispherical bowl of radius 10 cm contain water to a depth of h cm.

The volume of water in the bowl in terms of its depth h cm is given by

$$V = \frac{1}{3}\pi h^2 \left(30 - h\right) \text{ cm}^3.$$

Water is poured into the bowl at a rate of $2 \text{ cm}^3 \text{ s}^{-1}$.



i) If the radius of the water surface is r cm, show that $r = \sqrt{20h - h^2}$. **1**

When the depth of the water is 4 cm, find in terms of π :

ii) the rate of change of the water level. **2**

iii) the rate of change of the radius of the water surface. **3**

End of paper

CHS YEAR 12 EXTENSION 1 2017	TRIAL HSC SOLUTIONS

Multiple Choice Section:

1.B 2.A 3.C 4.C 5.D

6.A 7.D 8.C 9.B 10.D

Question 1.

The remainder is given by $P(2)$

$= 2(8) + 1 - 10(4) + 4 = -20$ ---------- $\boxed{B}$

Question 2.

$$x = \frac{2 \times -2 + 3 \times 8}{2 + 3} = 4$$ ---------- $\boxed{A}$

Question 3.

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \cdot \tan x}$$

$$= \frac{1 - \tan x}{1 + \tan x} = \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$ ----- $\boxed{C}$

Question 4.

$\angle RPQ = 22^\circ$ {$\angle$ at centre is twice $\angle$ at circumference}

$\angle ORM = 22^\circ$ {alt $\angle$'s, OR // PQ}

$\angle RMQ = 66^\circ$ {ext $\angle$ of triangle} -------- $\boxed{C}$

Question 5.

Using roots $(x = 0, 1, -2)$ and negative cubic

-------- $\boxed{D}$

Question 6.

$$x = \frac{1}{t},\ y = \frac{t}{t+1} \quad \rightarrow \quad t = \frac{1}{x}$$

$$\therefore \quad y = \frac{\frac{1}{x}}{\frac{1}{x} + 1} = \frac{1}{1 + x}$$

$\therefore \quad x \neq -1$ and $y \neq 0$ --------- $\boxed{A}$

Question 7.

$2x + y = 5 \quad \rightarrow \quad m_1 = -2$

$x - 3y = 1 \quad \rightarrow \quad m_2 = \frac{1}{3}$

$$\therefore \ \tan\theta = \left|\frac{-2 - \frac{1}{3}}{1 + (-2) \times \frac{1}{3}}\right| = 7$$

$\therefore \ \theta = 82^\circ$ ($NOTE$: $accept$ $81^\circ 52'$) ------- $\boxed{D}$

Question 8.

$$\int_0^{2k} \frac{1}{\sqrt{3 - x^2}}\, dx = \frac{\pi}{3}$$

$$\therefore \ \left[\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\right]_0^{2k} = \frac{\pi}{3}$$

$$\therefore \ \sin^{-1}\left(\frac{2k}{\sqrt{3}}\right) - \sin^{-1}(0) = \frac{\pi}{3}$$

$$\therefore \ \frac{2k}{\sqrt{3}} = \sin\frac{\pi}{3} \quad \rightarrow \quad \frac{2k}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\therefore \ k = \frac{3}{4}$ ------------ $\boxed{C}$

Question 9.

$$f(x) = x^3 - 12x \;\rightarrow\; f'(x) = 3x^2 - 12$$

$f'(x) = 0$ for stationary points

$$\therefore\; 3(x-2)(x+2) = 0 \;\rightarrow\; x = \pm 2$$

hence $-2 \le x \le 2$ ———————— $\boxed{B}$

Question 10.

$$v^2 = 2\int 2x^3 + 4x\,dx$$

$$\therefore\; v^2 = x^4 + 4x^2 + c$$

when $x = 2,\ v = 6$

$$\therefore\; 36 = 32 + c \;\rightarrow\; c = 4$$

$$\therefore\; v^2 = x^4 + 4x^2 + 4 = \left(x^2 + 2\right)^2$$

hence minimum speed when $x = 2$

$$\therefore\; v^2 = (4+2)^2 \;\rightarrow\; v = 6$$ ——————— $\boxed{D}$

Question 11 .

a) $$\frac{d}{dx}\left[\log_e(\ln x)\right] = \frac{1/x}{\ln x} = \frac{1}{x\ln x}$$

b) $$\lim_{\theta\to 0}\frac{5\theta}{\tan\frac{\theta}{2}} = 5\lim_{\theta\to 0}\frac{\frac{\theta}{2}}{\frac{1}{2}\tan\frac{\theta}{2}}$$

$$= \frac{5}{1/2} \times 1 = 10$$

c) $$y = be^{ax} \;\rightarrow\; y' = abe^{ax} \;\rightarrow\; y'' = a^2\left(be^{ax}\right)$$

$$\therefore\; y'' = a^2 y$$

d) $$\cos 2x = 2\cos^2 x - 1 \;\rightarrow\; \cos^2 2x = \frac{1}{2}(\cos 4x + 1)$$

$$\therefore\; I = \frac{1}{2}\int \cos 4x + 1\,dx$$

$$= \frac{1}{2}\left(\frac{1}{4}\sin 4x + x\right) + c$$

$$= \frac{1}{8}\sin 4x + \frac{1}{2}x + c$$

e) $$LHS = \frac{2\cos\theta}{\frac{1}{\sin\theta} - 2\sin\theta}$$

$$= \frac{2\cos\theta}{\frac{1-2\sin^2\theta}{\sin\theta}} = \frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta = RHS$$

f) CV1 and CV2: $x^2 - 3 = 0 \;\rightarrow\; x = \pm\sqrt{3}$

CV3: $x = 0$

On testing $-\sqrt{3} \le x < 0$ and $x \ge \sqrt{3}$, note $x \ne 0$

g) $$e^t - \frac{1}{e^t} = 2 \;\rightarrow\; \left(e^t\right)^2 - 2e^t - 1 = 0$$

$$\therefore\; e^t = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$$\therefore\; e^t = 1 + \sqrt{2} \text{ or } e^t = 1 - \sqrt{2}$$

$$\therefore\; t = \ln\left(1+\sqrt{2}\right) \text{ or } t = \ln\left(1-\sqrt{2}\right)$$

but since $\ln\left(1-\sqrt{2}\right)$ does not exist

then $t = \ln\left(1+\sqrt{2}\right)$ only .

Question 12.

a)i) Let $\sin\theta - \sqrt{3}\cos\theta = A\sin(\theta - \alpha)$

$$= A\cos\alpha\sin\theta - A\sin\alpha\cos\theta$$

Equating coefficients of $\cos x$ and $\sin x$ gives:

$A\cos\alpha = 1$ ———— $\boxed{1}$ and $A\sin\alpha = \sqrt{3}$ ———— $\boxed{2}$

$$\boxed{1}^2 + \boxed{2}^2 \;\rightarrow\; A^2\left(\cos^2\alpha + \sin^2\alpha\right) = 4$$

$$\therefore\; A^2 = 4 \;\rightarrow\; A = 2$$

Using $\boxed{1}$ with $A=2 \quad \rightarrow \quad \cos\alpha = \frac{1}{2}$

$\therefore \quad \alpha = \frac{\pi}{3}$

$\therefore \quad \sin\theta - \sqrt{3}\cos\theta = 2\sin\left(\theta - \frac{\pi}{3}\right)$

ii) minimum value $= -2$ since $-1 \le \sin A \le 1$

b) $\int_0^1 \frac{x^3}{1+x^8}\,dx \qquad u = x^4 - 1 \rightarrow du = 4x^3\,dx$

$x^4 = u+1 \rightarrow x^8 = (u+1)^2$

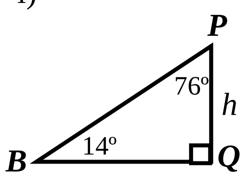
when $x=0, u=-1;\ x=1, u=0$

$$\therefore \quad I = \frac{1}{4}\int_{-1}^{0} \frac{1}{1+(u+1)^2}\,du$$

$$= \frac{1}{4}\left[\tan^{-1}(u+1)\right]_{-1}^{0}$$

$$= \frac{1}{4}\left(\tan^{-1}1 - \tan^{-1}0\right) = \frac{\pi}{16}$$

c) i)



$\tan 76° = \frac{BQ}{h}$

$BQ = h\tan 76°$

ii) Similarly using ΔPQA: $\quad AQ = h\tan 72°$

In ΔBQA: $\ BQ^2 + AQ^2 = 1500^2$

$\therefore \quad h^2\tan^2 76° + h^2\tan^2 72° = 1500^2$

$\therefore \quad h^2\left(\tan^2 76° + \tan^2 72°\right) = 1500^2$

$\therefore \quad h^2 = \frac{1500^2}{\tan^2 76° + \tan^2 72°} \quad \rightarrow \quad h = 297\,m$ (nearest metre)

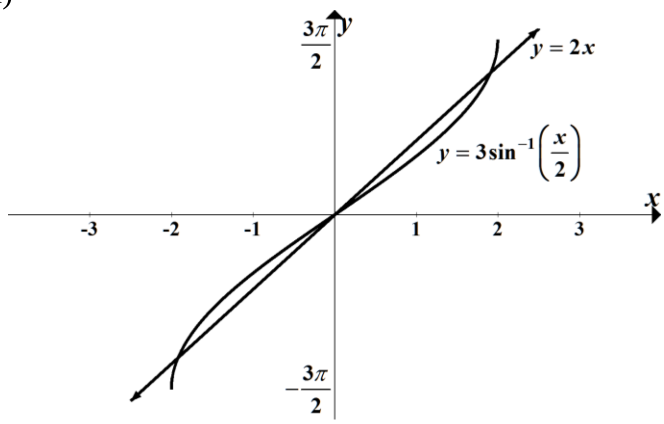
d) i) Function will be odd if $P(x) = -P(-x)$

$$-P(-x) = -\left[3\sin^{-1}\left(-\frac{x}{2}\right) - 2(-x)\right]$$

$$= -\left[-3\sin^{-1}\left(\frac{x}{2}\right) + 2(x)\right]$$

$$= 3\sin^{-1}\left(\frac{x}{2}\right) - 2(x) = P(x)$$

ii)



iii)

$P(x) = 3\sin^{-1}\left(\frac{x}{2}\right) - 2x \quad \rightarrow \quad P(1.9) = -0.04029$

$P'(x) = \frac{3}{\sqrt{4-x^2}} - 2 \quad \rightarrow \quad P'(1.9) = 2.8038$

$\therefore \quad x_2 = 1.9 - \frac{-0.04029}{2.8038} \approx 1.914(3dp)$

iv) zero (odd function has roots $-\alpha, 0, \alpha$)

Question 13.

a) $13 \times 6^n + 2$

When $n=1$: $13 \times 6^n + 2 = 13 \times 6^1 + 2$
$= 80 = 5 \times 16$

hence true for $n=1$

Assume true for $n = k$ and let
$13 \times 6^k + 2 = 5M \quad (M \text{ an integer})$ -----------**

Prove true for $n = k+1$

Hence $13 \times 6^{k+1} + 2 = 13 \times 6 \times 6^k + 2$

$$= 6 \times \left[13 \times 6^k\right] + 2$$
$$= 6 \times [5M - 2] + 2 \text{ using } **$$
$$= 30M - 10$$
$$= 5(6M - 2)$$

$\therefore$ true for $n = k + 1$

Hence by the principle of mathematical induction,

the result is true for all $n \geq 1$.

b) i) $x = 20\sqrt{3}\,t, \quad y = 20t - 5t^2$

Maximum height when $\dot{y} = 0$

$\therefore \dot{y} = 20 - 10t$

$\therefore 20 - 10t = 0 \quad \rightarrow \quad t = 2$

when $t = 2, \quad y = 20 \times 2 - 5 \times 2^2 = 20$ metres

ii) Range when it hits the ground $(y = 0)$

or twice the time to reach maximum height.

hence when $t = 4$.

$\therefore \quad R = 20\sqrt{3} \times 4 = 80\sqrt{3}$ metres

iii) $x = 20\sqrt{3}\,t \quad \rightarrow \quad t = \dfrac{x}{20\sqrt{3}}$

$$\therefore \; y = 20\left(\frac{x}{20\sqrt{3}}\right) - 5\left(\frac{x}{20\sqrt{3}}\right)^2$$

$$\therefore \; y = \frac{x}{\sqrt{3}} - \frac{x^2}{240}$$

c) i) Using the reference sheet, the equation of the normal at P is:

$x + py = ap^3 + 2ap$

and at N $x = 0$

$\therefore \; 0 + py = ap^3 + 2ap \; \rightarrow \; y = ap^2 + 2a$

$\therefore \; N\left(0, ap^2 + 2a\right)$

ii) $N\left(0, ap^2 + 2a\right);\; P\left(2ap, ap^2\right)$

$\therefore \; M\left(ap, ap^2 + a\right)$

From M $x = ap \; \rightarrow \; p = \dfrac{x}{a}$

$$\therefore \; y = a\left(\frac{x}{a}\right)^2 + a$$

$$\therefore \; y = \frac{x^2}{a} + a \; \rightarrow \; y = \frac{x^2 + a^2}{a}$$

$\therefore \; x^2 = a(y - a)$ which is a parabola

with focal length $\dfrac{a}{4}$

d) i) $v^2 = 5 + 4x - x^2$

The particle is at rest when $v^2 = 0$ $(ie\ v = 0)$.

$\therefore \; 5 + 4x - x^2 = 0$

$\therefore \; (x - 5)(x + 1) = 0 \quad \rightarrow \quad x = -1$ and $x = 5$

ii) amplitude $= 5 -$ centre of motion

$$= 5 - \frac{-1 + 5}{2} = 3$$

iii) $\ddot{x} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \dfrac{d}{dx}\left(\dfrac{1}{2}\left(5 + 4x - x^2\right)\right)$

$$\therefore \; \ddot{x} = 2 - x$$

iv) $\ddot{x} = -1^2(x - 2) \quad \rightarrow \quad n = 1$

$$\therefore \; P = \frac{2\pi}{n} = 2\pi$$

Question 14.

a) i) $\angle APB = 90°$ (angle in a semi-circle)

$\therefore \angle APQ = 90°$ (angle sum straight line)

Also $\angle AOQ = 90°$ (angle sum straight line)

$\therefore \angle APQ = 90° = \angle AOQ$

hence $AOPQ$ is a cyclic quad (∠'s in same segment)

ii) $\angle AQO = \angle APO$ {∠'s in same segment/cyc quad $AOPQ$}

Also $PSOB$ is a cyclic quad (opp ∠'s supplementary)

$\angle APO = \angle OBS$ {∠'s in same segment/cyc quad $PSOB$}

$\therefore \; \angle AQO = \angle OBS$

b) i) Since the sequence is geometric:

$$\frac{\gamma}{\beta} = \frac{\beta}{\alpha} \quad \rightarrow \quad \beta^2 = \alpha\gamma$$

ii) $\alpha\beta\gamma = -\frac{d}{a} = 27$

iii) Using (i) and (ii) : $\beta^3 = 27 \quad \rightarrow \quad \beta = 3$

also $\alpha + \beta + \gamma = -\frac{9}{2} \quad \rightarrow \quad \boxed{\alpha + \gamma = -\frac{15}{2}}$

and $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{27}{2}$

$\therefore \quad 3\alpha + \alpha\gamma + 3\gamma = -\frac{27}{2}$

$\therefore \quad 3(\alpha + \gamma) + \alpha\gamma = -\frac{27}{2} \quad \rightarrow \quad \boxed{\alpha\gamma = 9}$

$\therefore \quad \alpha + \frac{9}{\alpha} = -\frac{15}{2}$

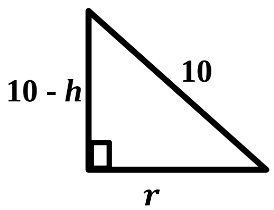
$\therefore \quad 2\alpha^2 + 15\alpha + 18 = 0$

$\therefore \quad (2\alpha + 3)(\alpha + 6) = 0$

$\therefore \quad \alpha = -\frac{3}{2}$ and $\alpha = -6$

hence the roots are $-6, -\frac{3}{2}, 3.$

c) i)



$r^2 + (10 - h)^2 = 10^2$

$r^2 = 100 - (10 - h)^2$

$= 20h - h^2$

$\therefore \quad r = \sqrt{20h - h^2}$

cii) $h = 4, \ \frac{dV}{dt} = 2, \ \frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi\left(30h^2 - h^3\right)$

$\therefore \quad \frac{dV}{dh} = \frac{\pi}{3}\left(60h - 3h^2\right)$

$\therefore \quad \frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$\therefore \quad 2 = \frac{\pi}{3}\left(60h - 3h^2\right) \times \frac{dh}{dt}$

and when $h = 4 \quad \rightarrow \quad \frac{dh}{dt} = \frac{1}{32\pi}\,\text{cm s}^{-1}$

iii) $r = \left(20h - h^2\right)^{\frac{1}{2}}$

$\therefore \quad \frac{dr}{dh} = \frac{1}{2}\left(20h - h^2\right)^{-\frac{1}{2}} \times (20 - 2h)$

$= \frac{10 - h}{\sqrt{20h - h^2}}$

Now $\frac{dr}{dt} = \frac{dr}{dh} \times \frac{dh}{dt}$

$= \frac{10 - h}{\sqrt{20h - h^2}} \times \frac{1}{32\pi}$

and when $h = 4 \quad \rightarrow \quad \frac{dr}{dt} = \frac{3}{128\pi}\,\text{cm s}^{-1}$